Indefinite integrals concepts

Core Concepts of Indefinite Integrals

1. Definition

An *indefinite integral* (or antiderivative) of a function $f(x)$ is another function $F(x)$ such that:

$$\frac{d}{dx}F(x) = f(x)$$

It is written as:

$$\int f(x)\,dx = F(x) + C$$

where C is the constant of integration (since differentiation of a constant is zero).

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2. Basic Rules

- **Linearity:**

$$\int [af(x) + bg(x)]\,dx = a\int f(x)\,dx + b\int g(x)\,dx$$

- **Constant Rule:**

$$\int k\,dx = kx + C$$

- **Power Rule (for $n \neq -1$):**

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

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3. Standard Integrals

- $\int \frac{1}{x}\,dx = \ln|x| + C$
- $\int e^x\,dx = e^x + C$
- $\int a^x\,dx = \frac{a^x}{\ln a} + C,\ (a > 0, a \neq 1)$
- $\int \sin x\,dx = -\cos x + C$
- $\int \cos x\,dx = \sin x + C$
- $\int \sec^2 x\,dx = \tan x + C$
- $\int \csc^2 x\,dx = -\cot x + C$
- $\int \sec x \tan x\,dx = \sec x + C$
- $\int \csc x \cot x\,dx = -\csc x + C$

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4. Methods of Integration

- **Substitution Method:** Useful when an expression can be simplified by a change of variable. Example: $\int 2x\cos(x^2)dx \ \rightarrow u = x^2$.
- **Integration by Parts:** Based on product rule of differentiation.

$$\int u\,dv = uv - \int v\,du$$

- **Partial Fractions:** For rational functions, decompose into simpler fractions.
- **Trigonometric Substitution:** For integrals involving $\sqrt{a^2 - x^2}, \sqrt{a^2 + x^2}, \sqrt{x^2 - a^2}$.

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5. Properties of Indefinite Integrals

- Differentiation of the result always leads back to the integrand.
- Multiple antiderivatives differ only by a constant.
- Used for general solution of differential equations.

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6. Example Problems

1. $\int (3x^2 - 4x + 5)\,dx = x^3 - 2x^2 + 5x + C$
2. $\int e^{2x}dx = \frac{1}{2}e^{2x} + C$
3. $\int \frac{1}{1+x^2}dx = \tan^{-1}(x) + C$
