W0701 Maths : Class XII KL095 ①
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Integrals

1. $= \int (x - x^{-2})\,dx = \frac{x^2}{2} - \frac{(-1)}{x} + C = \frac{x^2}{2} + \frac{1}{x} + C$

2. $= \int \left(\frac{\sin x}{\cos x}\right)^2 dx = \int \tan^2 x\,dx = \int (\sec^2 x - 1)\,dx$

$= \tan x - x + c.$

3. $= \int \frac{x^2(x-1) + (x-1)}{(x-1)}\,dx = \int \frac{(x-1)(x^2+1)}{(x-1)}\,dx$

$= \int (x^2+1)\,dx = \frac{x^3}{3} + x + c.$

4. put $\sqrt{x} = t \Rightarrow x = t^2$

$dx = 2t\,dt$ | put $\tan t = k$, $\sec^2 t\,dt = dk$

$GI = 2\int \tan^4 t \cdot \sec^2 t\,dt = 2 \cdot \int k^4\,dk = 2 \cdot \frac{(\tan\sqrt{x})^5}{5} + c.$

5. $\int \sec x \times \frac{(\sec x + \tan x)}{(\sec x + \tan x)}\,dx = \int \frac{(\sec^2 x + \sec x \tan x)}{\sec x + \tan x}\,dx$

$= \int \frac{(\sec x \tan x + \sec^2 x)}{(\sec x + \tan x)}\,dx = \log|\sec x + \tan x| + c.$

6. $\int_{-1}^{2} |x^3 - x|\,dx = -\int_{-1}^{0} x^3 - x\,dx + \int_{0}^{2} x^3 - x\,dx = -\left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{0}^{2}$

$= -\left[0 - \left(\frac{1}{4} - \frac{1}{2}\right)\right] + \left[\frac{4^{\cancel{2}}}{\cancel{4}} - \frac{2^{\cancel{2}}}{\cancel{2}} - 0\right] = \frac{1}{4} + 2 = \left(\frac{7}{4}\right)$

7. $I = \int_0^{\pi} \frac{(\pi - x)(-\tan x)}{(-\sec x + (-\tan x))} dx$ & also let $I = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$

$I = \int_0^{\pi} \frac{\pi \tan x}{\sec x + \tan x} dx - I$

$\left[\because \sec x + \tan x = \frac{1}{\sec x - \tan x}\right.$
$\Rightarrow \sec^2 x - \tan^2 x = 1$

$\Rightarrow I = \frac{\pi}{2} \cdot \int_0^{\pi} \cancel{\sin x} \tan x (\sec x - \tan x) dx$

$= \frac{\pi}{2} \left\{ \int_0^{\pi} \sec x \tan x \, dx - \int_0^{\pi} (\sec^2 x - 1) dx \right\}$

$= \frac{\pi}{2} \left\{ \frac{1}{\cos(\pi)} - \frac{1}{\cos 0} - (\tan \pi - \tan 0) + (\pi - 0) \right\}$

$= \frac{\pi}{2} \left\{ -2 + \pi \right\} = \frac{\pi}{2}(\pi - 2)$

8. $= \int \frac{(\sin^4 x + \cos^4 x)(\sin^4 x - \cos^4 x) dx}{\sin^4 x + \cos^4 x - 2\sin^2 x \cos^2 x}$

$= \int \frac{(\sin^4 x + \cos^4 x)(\sin^4 x - \cos^4 x)}{(\sin^2 x - \cos^2 x)} dx$

$= \int (\sin^4 x + \cos^4 x)(\sin^2 x + \cos^2 x) \, dx$

$= \int (\sin^4 x + \cos^4 x) \, dx = \frac{(\sin x)^5}{5} - \frac{(\cos x)^5}{5} + c.$

9. $$\int \frac{dx}{\sqrt{\sin^3 x \sin(x+\alpha)}}$$

$$= \int \frac{\operatorname{cosec} x \, dx}{\sqrt{\sin^2 x \cos\alpha + \sin x \cos x \sin\alpha}}$$

$$= \int \frac{\operatorname{cosec}^2 x \, dx}{\sqrt{\cos\alpha + \cot x \sin\alpha}}$$

put $\cos\alpha + \cot x \cdot \sin\alpha = t^2$

$+ \operatorname{cosec}^2(x)\, dx = \frac{2t\,dt}{-\sin\alpha}$

$$= \frac{-1}{\sin\alpha} \int \frac{2\cancel{t}\,dt}{\cancel{t}}$$

$$= \frac{-2}{\sin\alpha}\left(\sqrt{\cos\alpha + \cot x \cdot \sin\alpha}\right) + c.$$

10. $$= \int \sqrt{\cot x}\, dx + \int \sqrt{\tan x}\, dx$$

~~$\cot x = a^2$~~

~~$\tan x = b^2$~~

$$= \int \sqrt{\frac{\sin x}{\cos x}}\, dx + \int \sqrt{\frac{\cos x}{\sin x}}\, dx$$

~~$\cos x = t^2$~~

~~$-\sin x\, dx = 2t\, dt$~~

$$= \int \frac{\sqrt{2}\,\sin x + \cos x}{\sqrt{2 \sin x \cos x + 1 - 1}}\, dx$$

~~$\sqrt{\sin x}\, dx = \sqrt{-2t}\, dt$~~

~~$\sin x \cos x = t^2$~~

$$= (\sqrt{2}) \int \frac{(\sin x + \cos x)\, dx}{\sqrt{1 - (\cos^2 x + \sin^2 x - 2 \sin x \cos x}}$$

~~$(\cos^2 x - \sin^2 x) dx = 2t\, dt$~~

$$= (\sqrt{2}) \int \frac{(\sin x + \cos x)\, dx}{\sqrt{1 - (\sin x - \cos x)^2}}$$

put $\sin x - \cos x = t$

$(\sin x + \cos x)\, dx = dt$

$$= \sqrt{2} \int \frac{dt}{\sqrt{1-t^2}} = \sqrt{2}. \sin^{-1}(t)$$

$$= \sqrt{2} \sin^{-1}(\sin x - \cos x) + c.$$