CHAPTER 9: Differential Equations

Order and Degree of a Differential Equation, General and Particular Solutions of a Differential Equation

1. 2, as there are two arbitrary constants and we have to differentiate twice.
2. One, as there is one arbitrary constant.
3. Three, as equation cannot be represented as polynomial of derivatives.
4. (*b*), as degree $p = 1$ and order $q = 3$

   $\therefore\ 2p - 3q = 2 - 9 = -7$
5. (*c*), as differential equation is

   $$1 + 3\frac{dy}{dx} + 3\left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^3 = \left(\frac{dy}{dx}\right)^2.$$

   Exponent of highest order derivative is 3.
6. (*d*), as equation cannot be represented as a polynomial of derivatives.
7. (*b*), as centre is arbitrary (h, k), two arbitrary constants so we have to differentiate twice to eliminate h, k

   $\therefore$ order is 2.
8. False, as equation can be written as

   $$1 + \frac{d^2y}{dx^2} = \left(x + \frac{dy}{dx}\right)^2.$$ Further it can be written as a polynomial of derivatives.
9. Degree 1
10. Degree 3.

11. Given $\dfrac{4\left(\dfrac{d^2y}{dx^2}\right)^3}{\dfrac{d^3y}{dx^3}} + \dfrac{d^3y}{dx^3} = x^2 - 1$

$$\Rightarrow \quad 4\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{d^3y}{dx^3}\right)^2 = (x^2 - 1)\left(\frac{d^3y}{dx^3}\right)$$

Order of differential equation $(m) = 3$

Degree of differential equation $(n) = 2$

12. Highest order derivative is $\frac{dy}{dx}$. Hence, order of differential equation is 1.

Equation cannot be written as a polynomial in derivatives. Hence, degree is not defined.

Formation of a Differential Equation Corresponding to A given Function

13. (*c*), as general equation of line through origin is

$$y = mx \quad \Rightarrow \quad \frac{dy}{dx} = m$$

Substituting in (*i*), we get

$$y = \frac{dy}{dx} \cdot x \quad \Rightarrow \quad x\,dy - y\,dx = 0$$

14. Consider $\quad y = ae^{bx+5}$

On differentiating both sides, w.r.t, x, we get

$$\frac{dy}{dx} = abe^{bx+5} = by \qquad \text{...}(i)$$

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = b \cdot \frac{dy}{dx} \qquad \text{...}(ii)$$

From (*i*) and (*ii*), eliminating b, we get

$y \cdot \dfrac{d^2y}{dx^2} = \left(\dfrac{dy}{dx}\right)^2$ as required equation.

15. General equation of parabola is $y^2 = 4ax$ ...(*i*)

Differentiating, we get $2yy' = 4a$

$\Rightarrow \quad yy' = 2a.$

Substituting in (*i*), we get $y^2 = 2xyy'$.

16. $\quad x^2 = 4ay$

$\Rightarrow \quad 2x = 4ay' \Rightarrow \dfrac{x^2}{2x} = \dfrac{4ay}{4ay'}$

$\Rightarrow \quad xy' - 2y = 0$ is the required equation.

Solutions of Differential Equations with Variables Seperable

17. $-\dfrac{1}{e^y} = \dfrac{1}{2}e^{2x} + C$, as $\dfrac{dy}{dx} = e^{2x+y} = e^{2x} \cdot e^y$

$$\Rightarrow \quad e^{-y}dy = e^{2x}dx$$

$$\Rightarrow \quad -e^{-y} = \frac{1}{2}e^{2x} + C$$

18. $\quad \int e^y\,dy = \int (e^x + x^3)dx$

$$\Rightarrow \quad e^y = e^x + \frac{x^4}{4} + C$$

19. $\int \dfrac{dy}{y} = \int \tan x\,dx$

$\Rightarrow \quad \log|y| = \log|\sec x| + \log C$

$\Rightarrow \quad y = C\sec x \qquad \text{...}(i)$

Given $\quad y = 1, x = 0$

$\Rightarrow \quad 1 = C\sec 0$

$\Rightarrow \quad C = 1$

$\therefore$ solution is $y = \sec x$ [from (*i*)]

20. $\int (2 - y)dy = \int (x + 1)\,dx$

$$\Rightarrow \quad 2y - \frac{y^2}{2} = \frac{x^2}{2} + x + C$$

is the required solution.