Gauss Law

AND ITS APPLICATIONS

Electrostatics, Physics, 12th Std

Electric flux, Electric flux for closed surfaces, Gauss Law, Applications.

References:
SL Arora Physics, Physics State Board Textbook,
E. Ramanathan - Sai Tech Lecture Notes.

CONCEPT MNEMONIC: ISLCLNFAP

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31/8/2019

Concept Map

Gauss law and its applications

1. Will not hold in case of any departure from inverse square law
2. applicable to any field obeying inverse square law
3. inverse square dependance on distance in Coulomb's law
4. charge 'q' in theorem, contained within the closed surface
5. sum of all charges located anywhere inside the closed surface
6. net flux due to closed surface, inside & outside, is zero
7. If net charge enclosed by a closed surface = 0, $\phi_E = \frac{q}{\varepsilon_0} = 0$
8. closed surface
9. Electric flux through a closed surface S, $\phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$
10. For medium of absolute permittivity ε or dielectric constant K, $\oint \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon} = \frac{q}{K\varepsilon_0}$

Terms, Definitions and Symbols:

ELECTRIC FLUX:

The number of electric field lines crossing a given area kept normal to the electric field lines is called electric flux. It is usually denoted by the Greek letter ϕ. and its unit is Nm^2C^{-1}! Electric flux is a scalar quantity and it can be positive or negative

GAUSS LAW:

Gauss law states that if a charge 'Q' is enclosed by an arbitary closed surface, then the total electric flux ϕ_E through the closed surface is $\phi_E = \oint \vec{E}.d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$

DATA TABLE, EQUATIONS AND FORMULAE:

ELECTRIC FLUX FOR UNIFORM ELECTRIC FIELD IN A REGION OF SPACE:

$$\phi_E = EA$$

$A \rightarrow$ Area
$E \rightarrow$ Electric field

AREA 'A' IS KEPT PARALLEL TO UNIFORM FIELD, THEN,

$$\phi_E = 0$$

IF AREA IS INCLINED AT AN ANGLE θ, THEN,

$$\phi_E = (E \cos\theta) A$$

FOR UNIFORM ELECTRIC FIELD, ELECTRIC FLUX,

$$\phi_E = \vec{E} \cdot \vec{A} = EA \cos\theta$$

ELECTRIC FLUX IN A NON UNIFORM ELECTRIC FIELD AND AN ARBITARY SHAPED AREA,

$$\phi_E = \sum_{i=1}^{n} \vec{E_i} \cdot \Delta\vec{A_i}$$

By taking limit $\Delta\vec{A_i} \rightarrow 0$, summation becomes integration. And so,

$$\phi_E = \int \vec{E} \cdot d\vec{A}$$

TOTAL ELECTRIC FLUX FOR CLOSED SURFACE,

$$\phi_E = \oint \vec{E} \cdot d\vec{A}$$

TOTAL FLUX THROUGH THE CLOSED SURFACE OF THE SPHERE,

$$\phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E dA \cos\theta$$

ELECTRIC FIELD OF POINT CHARGE DIRECTED RADIALLY OUTWARD,

$$\phi_E = \oint E dA \qquad (\text{since } \cos 0° = 1)$$

E is uniform on the surface of the sphere,

$$\phi_E = E \oint dA.$$

GAUSS LAW:

$$\phi_E = \frac{Q}{\varepsilon_0}$$

$Q \rightarrow$ charge
$\varepsilon_0 \rightarrow$ permittivity of free space.

GAUSS LAW FOR CLOSED SURFACES:

$$\phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

TOTAL ELECTRIC FLUX IN CLOSED SURFACES (EXPANDED)

$$\phi_E = \oint \vec{E} \cdot d\vec{A}$$

$$= \int_{\substack{\text{curved}\\ \text{surface}}} \vec{E} \cdot d\vec{A} + \int_{\substack{\text{Top}\\ \text{surface}}} \vec{E} \cdot d\vec{A} + \int_{\substack{\text{Bottom}\\ \text{surface}}} \vec{E} \cdot d\vec{A}$$

FOR CYLINDRICAL GAUSSIAN SURFACE,

$(\because)$ $Q_{encl} = \lambda L$

$$E \int_{\substack{\text{curved}\\ \text{surface}}} dA = \frac{\lambda L}{\varepsilon_0}$$

$\lambda \rightarrow$ linear charge density

Here $\phi_E = \int_{\substack{\text{curved}\\ \text{surface}}} dA$ = total area of the curved surface $= 2\pi r L$.

Substituting,

$$E \cdot 2\pi r L = \frac{\lambda L}{\varepsilon_0}$$

$$E = \frac{1}{2\pi\varepsilon_0} \frac{\lambda}{r}$$

In vector form $\vec{E} = \frac{1}{2\pi\varepsilon_0} \frac{\lambda}{r} \hat{r}$

ELECTRIC FIELD DUE TO CHARGED INFINITE PLANE SHEET:

$$\phi_E = \oint \vec{E} \cdot d\vec{A}$$

$$= \int_{\substack{\text{curved}\\ \text{surface}}} \vec{E} \cdot d\vec{A} + \int_{P} \vec{E} \cdot d\vec{A} + \int_{P'} \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

WHEN ELECTRIC FIELD IS PERPENDICULAR TO AREA AND PARALLEL TO SURFACE,

$$\phi_E = \int_P E\,dA + \int_{P'} E\,dA = \frac{Q_{encl}}{\varepsilon_0}$$

Since magnitude of electric field at both surfaces is uniform, E is taken out of the integration and Q_{encl} is given by $Q_{encl} = \sigma A$, we get

$$2E \int_P dA = \frac{\sigma A}{\varepsilon_0}$$ $\sigma \rightarrow$ surface charge density

The total area of surface either at P or P'

$$\int_P dA = A$$

Hence $2EA = \frac{\sigma A}{\varepsilon_0}$ (or) $E = \frac{\sigma}{2\varepsilon_0}$

In vector form, $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$

ELECTRIC FIELD DUE TO TWO PARALLEL CHARGED INFINITE SHEETS:

$$E_{inside} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

ELECTRIC FIELD DUE TO A UNIFORMLY CHARGED SPHERICAL SHELL:

Case (a):

AT A POINT OUTSIDE THE SHELL ($r > R$)

$$\oint_{\text{Gaussian surface}} \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

Since $\vec{E}$ is same at all points,

$$E \oint_{\substack{\text{gaussian} \\ \text{surface}}} dA = \frac{Q}{\varepsilon_0}$$

But $\oint_{\substack{\text{Gaussian} \\ \text{surface}}} dA$ = total area of gaussian surface

$= 4\pi r^2$. Substituting,

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \quad (\text{or}) \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

In vector form, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$

Case (b): AT A POINT ON THE SURFACE OF THE SPHERICAL SHELL ($r = R$)

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{r}$$

Case (c): AT A POINT INSIDE THE SPHERICAL SHELL ($r < R$)

$$\oint_{\substack{\text{Gaussian} \\ \text{surface}}} \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

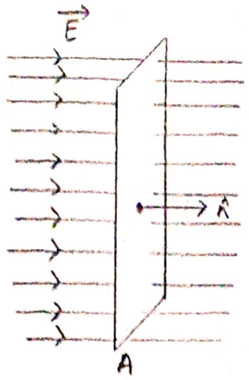
$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

Since gaussian surface encloses no charge, so $Q = 0$. Equation becomes,

$E = 0 \quad (r < R)$

INFOGRAPHICS:

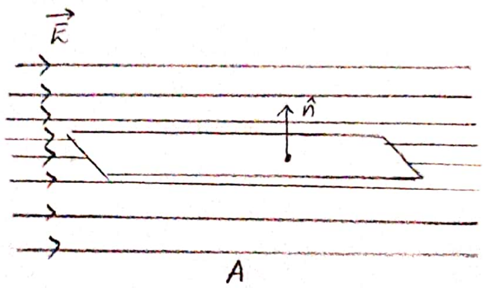
THE ELECTRIC FLUX FOR UNIFORM ELECTRIC FIELD

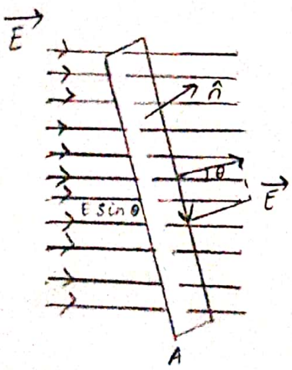


(a) Electric flux = EA

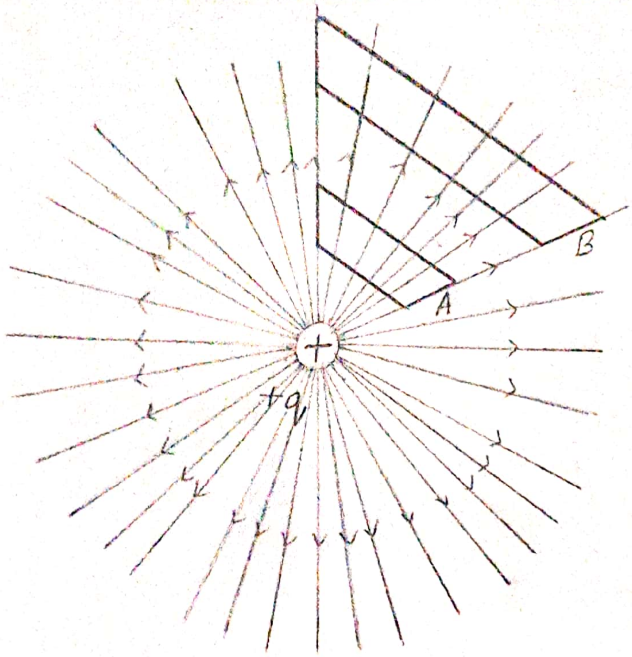
(b) Electric flux = 0

Here $\vec{A} = A\hat{n}$

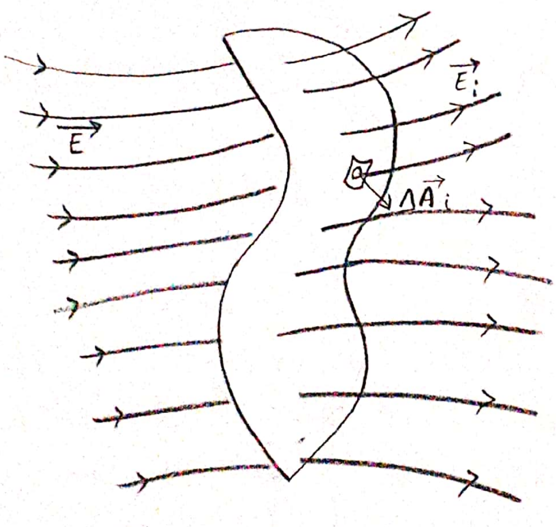




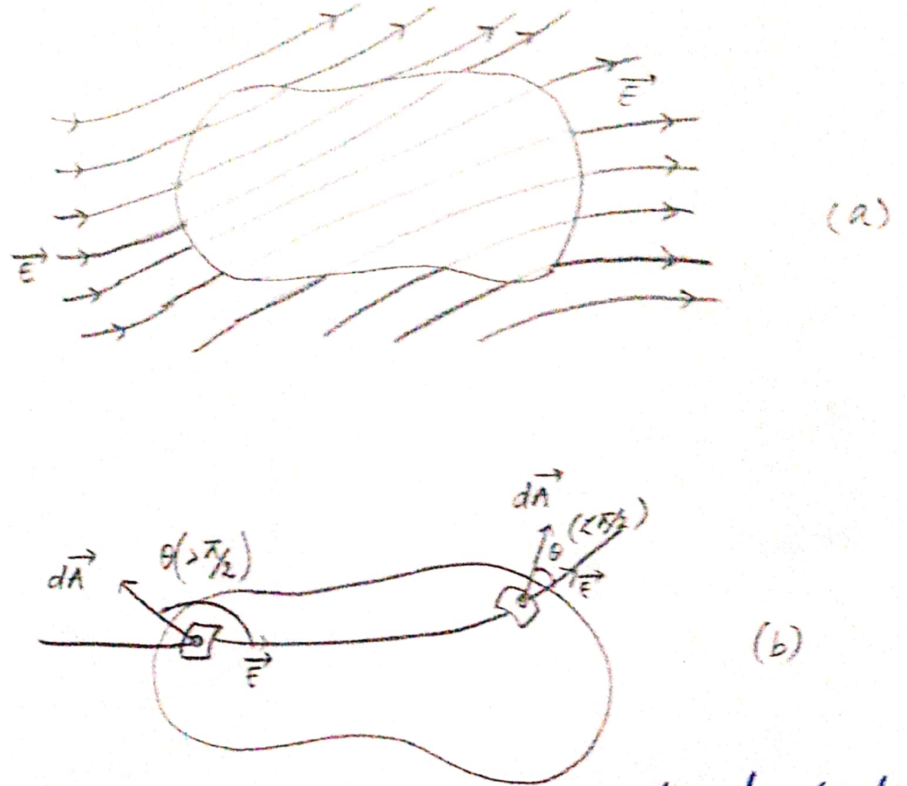
(c) Electric flux = $(E \cos\theta)\, A$



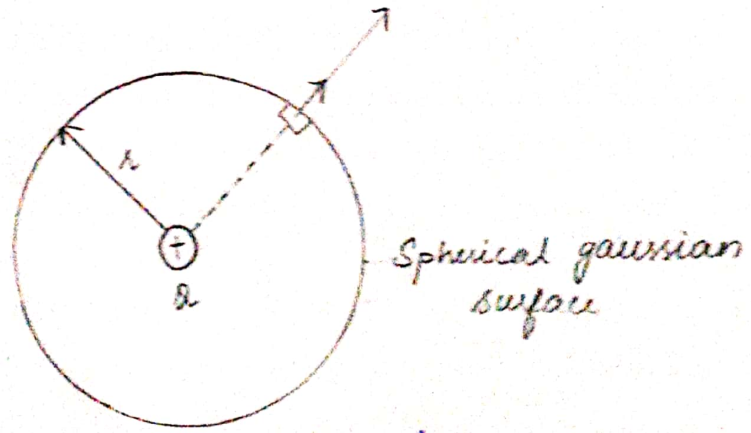
Electric flux



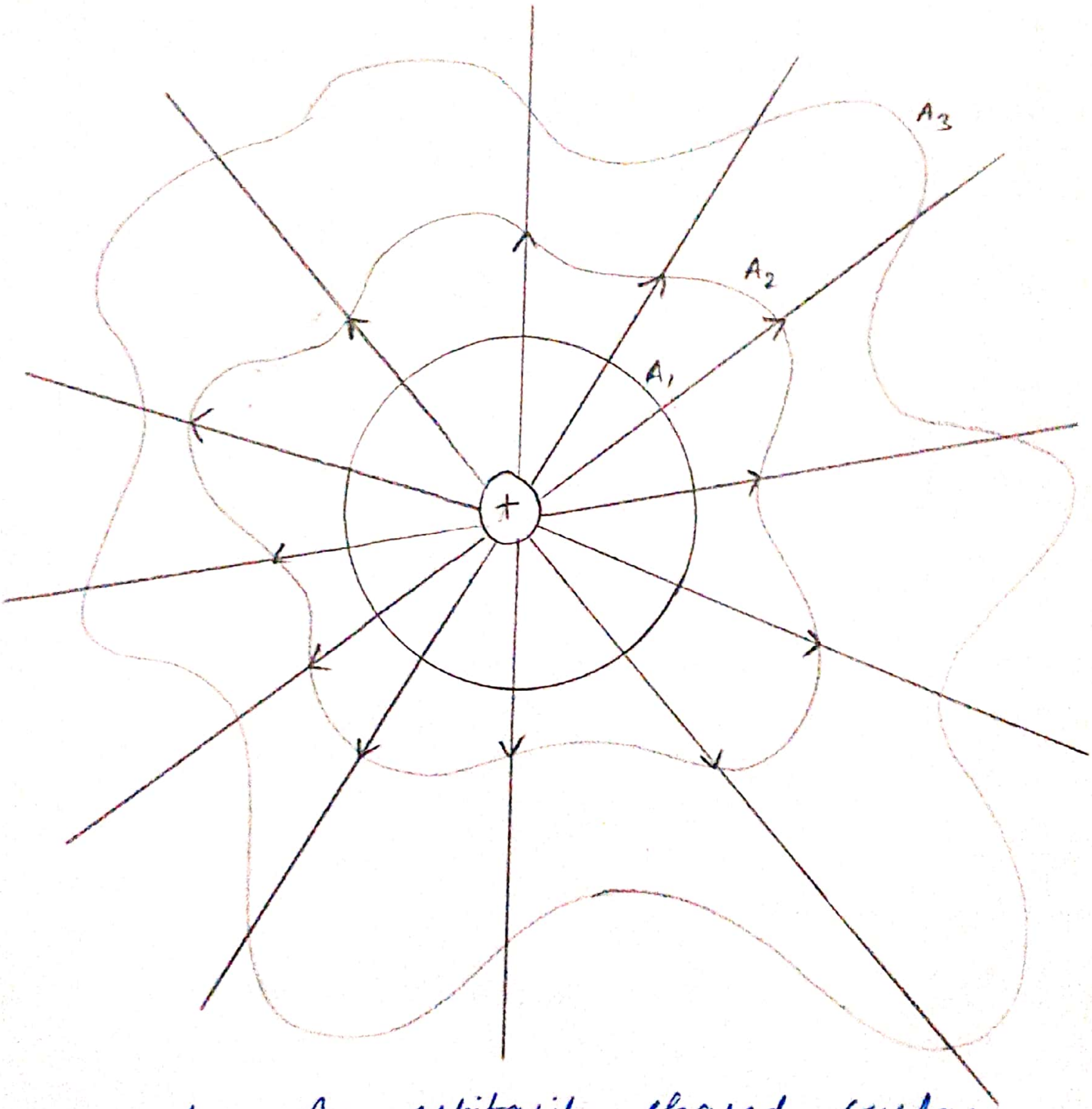
Electric flux for non-uniform electric field.



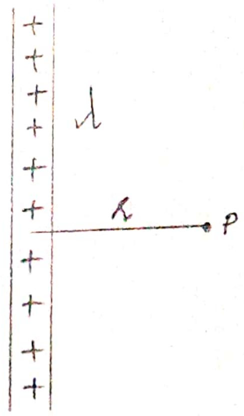
(a) & (b) Electric flux over a closed surface

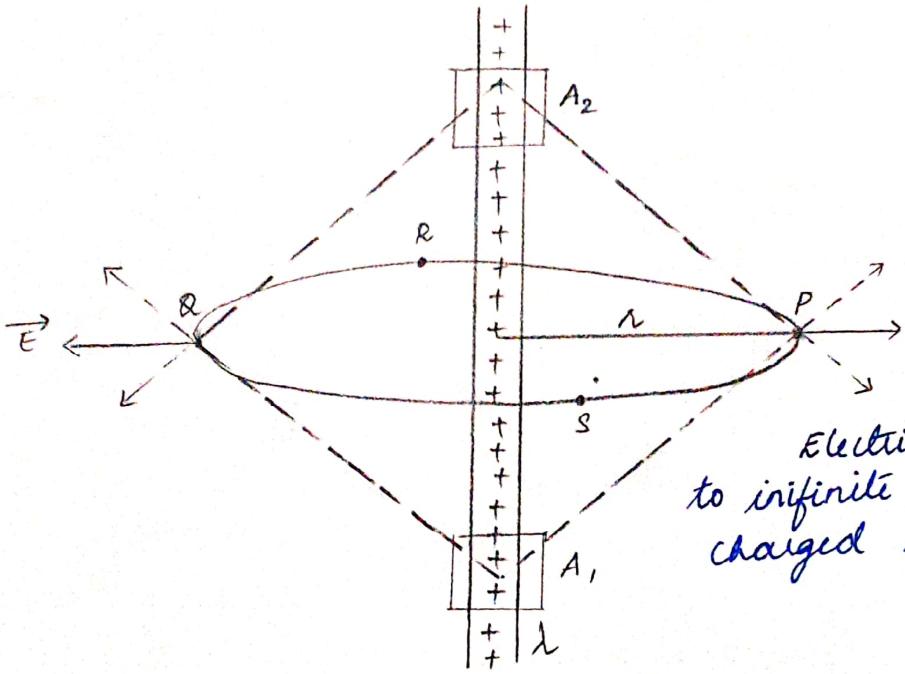


Total electric flux of point charge.

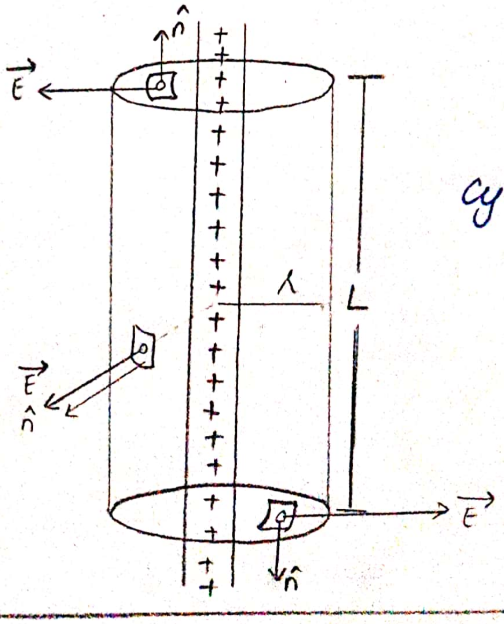


gauss law for arbitarily shaped surface

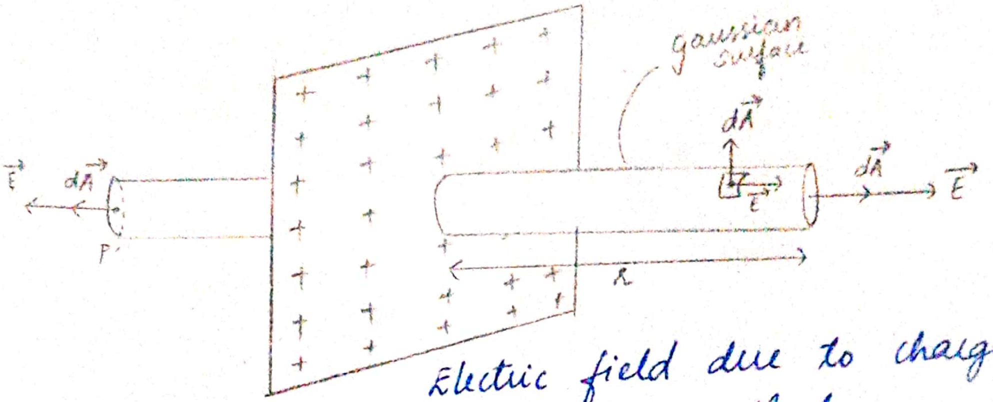




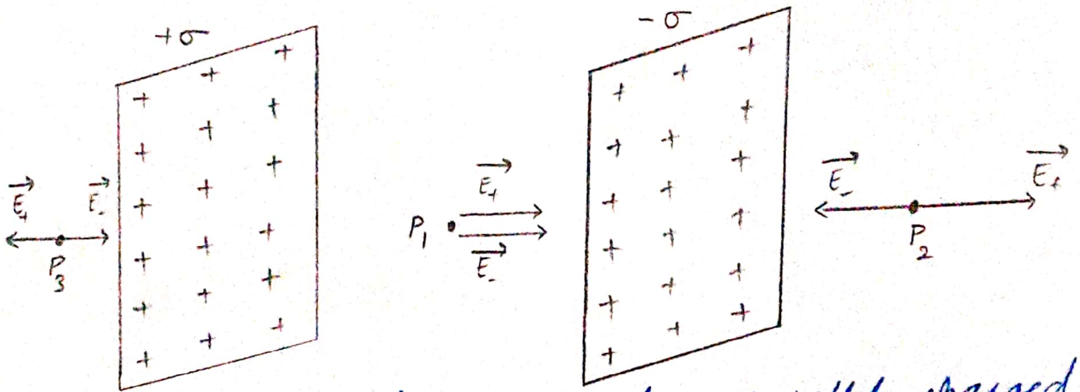
Electric field due to infinite long charged wire.



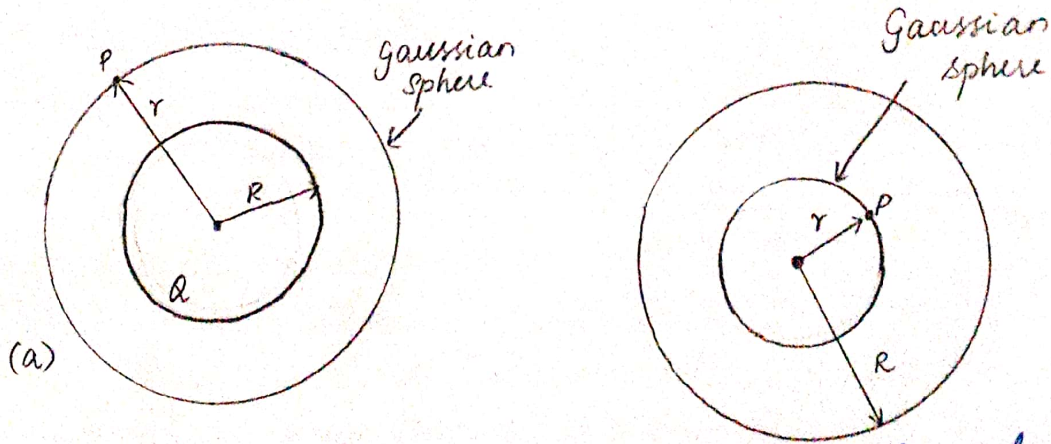
cylindrical gaussian surface.



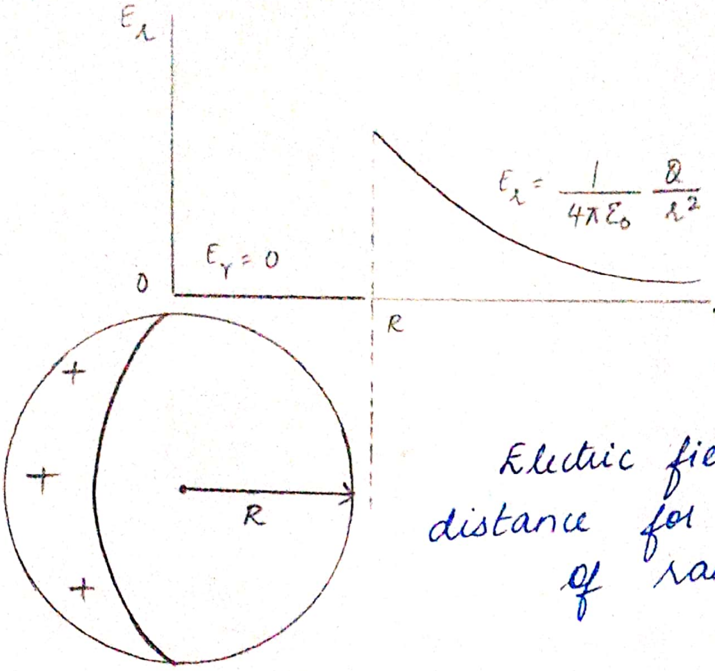
Electric field due to charged infinite planar sheet.



Electric field due to two parallel charged sheets



The electric field due to a charged spherical shell.



Electric field versus distance for a spherical shell of radius R.

MNEMONICS

NEEN - Electric flux is **NE**gative if the electric field lines **EN**ter the closed surface

POLE - Electric flux is **PO**sitive if the electric field lines **LE**ave the closed surface

DECPONCFA - Total electric flux through the closed surface **D**epends only on the charges **E**nclosed by the surface and the **C**harges **P**resent **O**utside the surface will **N**ot **C**ontribute to the **F**lux and the shape of the closed surface which can be chosen **A**rbitarily

ILCI - The total electric flux is Independent of the Location of the Charges Inside the closed surface.

ISGS - Imaginary Surface is called as Gaussian Surface.

SOG α TOCC, KOS - The Shape Of the Gaussian surface to be chosen depends on the Type Of Charge Configura-tion and the kind Of Symmetry

GSCPTDC - The Gaussian Surface Cannot Pass Through any Discrete Charge

GSCPTCCD - The Gaussian Surface Can Pass Through Continuous Charge Distributions.

AFOCL - Gauss law is Another Form Of Coulomb's Law

Applications of Gauss Law:

EFILCW - Electric Field due to an Infinitely Long Charged Wire.

EFCIPS - Electric Field due to Charged Infinite Plane Sheet

EFTPCIS - Electric Field due to Two Parallel Charged Infinite Sheets

EFUCSS - Electric Field due to a Uniformly Charged Spherical Shell.