COMPLEX NUMBERS

UNIT : COMPLEX NUMBERS

SUBJECT : MATHEMATICS

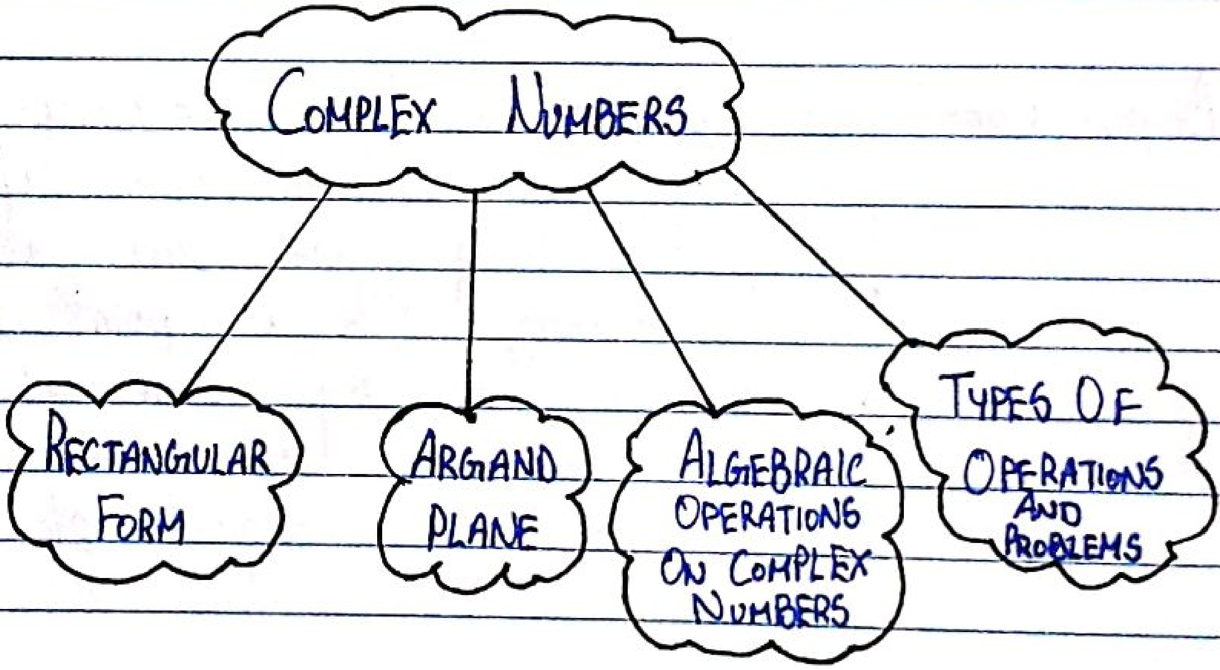
CLASS LEVEL : XII

STATE BOARD TNSCERT

CONCEPT ABSTRACT

A complex number is a quantity of the of the form $a+ib$ where a and b are real numbers and i represents the unit imaginary number equal to the positive square root of -1. This concept deals with various forms and operations of complex numbers.

CONCEPT MAP :



TERMS DEFINITIONS AND SYMBOLS

Rectangular form of a complex Numbers: A complex number is of the form $x+iy$ (or $x+yi$), where x and y are real numbers. x is called the real part and y is called the imaginary part of the complex number.

Relation between z_1 and z_2: Two complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are said to be equal if and only if $Re(z_1) = Re(z_2)$ and $Im(z_1) = Im(z_2)$. That is $x_1 = x_2$ and $y_1 = y_2$.

Argand Plane: A complex number $z = x+iy$ is uniquely determined by an ordered pair of real numbers (x,y). By this way we can associate a complex number with a point (x,y) in a coordinate plane. The xy plane which represents with y as imaginary axis and x as real axis is called as argand plane.

(i) Addition of Complex Numbers:

If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ where x_1, x_2, y_1 and $y_2 \in \mathbb{R}$, then we define

$$z_1 + z_2 = (x_1 + iy_1) + (x_2 + iy_2)$$
$$= (x_1 + x_2) + i(y_1 + y_2)$$

$$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2).$$

(ii) Scalar Multiplication of Complex Numbers:

If $z = x + iy$ and $k \in \mathbb{R}$, then we define

$$kz = (kx) + (ky)i$$

In particular $0z = 0$, $1z = z$ and $-1z = -z$.

(iii) Subraction of Complex Numbers:

The difference $z_1 - z_2$ can be drawn as a position vector whose initial point is the origin and terminal point is $(x_1 - x_2, y_1 - y_2)$.

We define
$$z_1 - z_2 = z_1 + (-z_2)$$
$$= (x_1 + iy_1) + (-x_2 - iy_2)$$
$$= (x_1 - x_2) + i(y_1 - y_2)$$
$$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2).$$

(iv) Multiplication of Complex Numbers:

The multiplication of complex numbers z_1 and z_2 is defined as:

$$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2)$$
$$= (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$$

$$z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1).$$

TYPES OF PROBLEMS:

- Addition, Multiplication, Subraction and Scalar Multiplication of Complex Numbers.

EXAMPLES:

1. Evaluate zw if $z = 5-2i$ and $w = -1+3i$

$$zw = (5-2i)(-1+3i)$$
$$= -5 + 15i + 2i - 6i^2 \quad [\because i^2 = -1]$$
$$= -5 + 17i + 6$$
$$= 1 + 17i$$

- Finding values of real numbers x and y from given equations.

- Plotting the given complex numbers on the argand plane.

EXAMPLE:

Find the value of real numbers x and y, if the complex number $(2+i)x+(1-i)y+2i-3$ and $x+(-1+2i)y+1+i$ are equal.

GIVEN:

$(2+i)x+(1-i)y+2i-3 = x+(-1+2i)y+1+i$

TO FIND:

Values of real numbers x and y.

REFERENCE:

Relation between z_1 and z_2.

FORMULA:

$$z_1 = z_2 \Rightarrow x_1+iy_1 = x_2+iy_2.$$

SUBSTITUITION:

$(2+i)x+(1-i)y+2i-3 = x+(-1+2i)y+1+i$

$(2+i-1)x+(1-i+\cancel{1}-2i)y+2i-i-3-1=0$

$(1+i)x+(2-3i)y+i-4=0.$

$(1+i)x + (2-3i)y + i - 4 = 0$.

Grouping real and imaginary parts together,

$x + ix + 2y - 3iy + i - 4 = 0$

$x + 2y - 4 + ix - 3iy + i = 0$

$x + 2y - 4 = 0$ — ①

$x - 3y + 1 = 0$ — ②

On solving we get

$x + 2y - 4 = 0$ — ① $\times 1$

$x - 3y + 1 = 0$ — ② $\times -1$

$\cancel{x} + 2y = 4$

$-\cancel{x} + 3y = 1$

$5y = 5$

$5y = 5$

$\boxed{y = 1}$

Substituting the value of y in ① we get

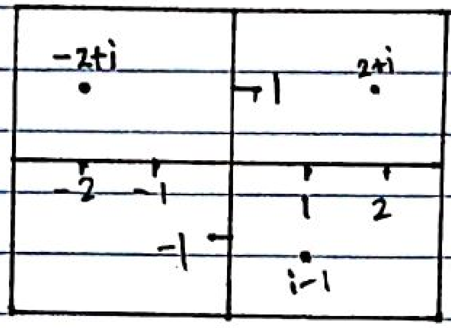
$x + 2 = 4$

$x = 4 - 2$

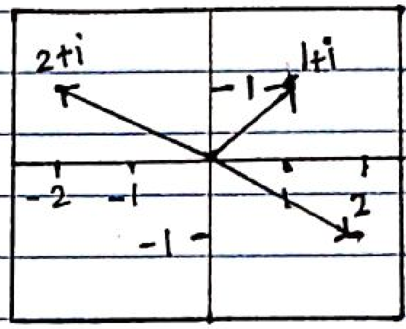
$\boxed{x = 2}$

$\therefore$ The value of x and y are 2 and 1 respectively.

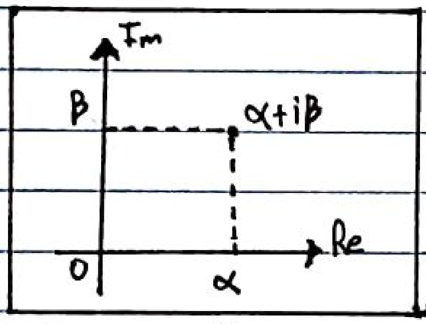
InfoGraphics :



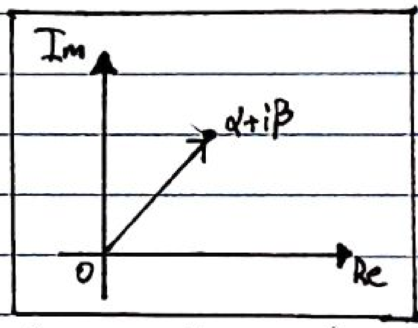
Complex Numbers As Points



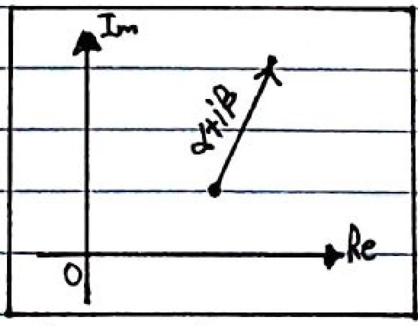
Complex Numbers As Vectors



Complex Numbers As A Point



Complex Number By A Position Vector Pointing From Origin To The Point



Complex Number As A Vector