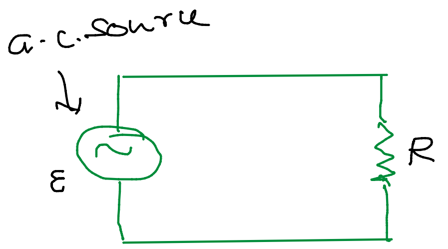
A.C. Voltage applied to a resistor

18 July 2019 10:38



Ohm's law

$$\boxed{V = IR} \quad \text{—(2)}$$

$V = V_m \sin\omega t$

$IR = V_m \sin\omega t$ (from (1) & (2))

$I = \dfrac{V_m \sin\omega t}{R}$

$I = I_m \sin\omega t$

$I_m \cancel{\sin\omega t} = \dfrac{V_m}{R} \cancel{\sin\omega t}$

$I_m = \dfrac{V_m}{R}$

R = Resistance (constant)

V_m = voltage amplitude (or) peak voltage

I_m = peak current

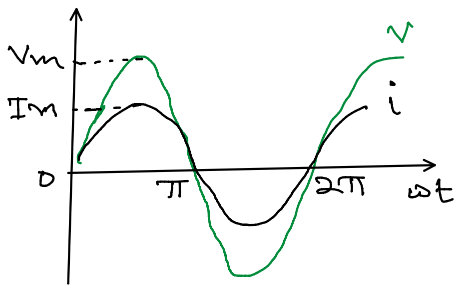
$$\boxed{V = V_m \sin\omega t} \quad \text{—(1)}$$

where

V = a.c. voltage

V_m = Voltage amplitude (or) peak voltage

ω = angular frequency



I_m	= peak current
V_m	= peak voltage
V	= voltage
I	= current

Joule's heating

$H = I^2 R$

H = Heat dissipated

Power dissipated

$P = I^2 R$

$P = I_m^2 \cdot \sin^2\omega t \; R$

$I = I_m \cdot \sin\omega t$

$P = I_m^2 \cdot R \cdot \sin^2\omega t$

$\bar{P} = \langle I^2 R \rangle$

$\bar{P} = I_m^2 R \langle \sin^2\omega t \rangle$

$\bar{P}$ = average power

$\langle \; \rangle$ average of the quantity

I_m^2, R = constants

$\sin^2\theta = \dfrac{(1-\cos 2\theta)}{2}$

$\bar{P} = I_m^2 R \left\langle \dfrac{1-\cos 2\omega t}{2} \right\rangle$

NOTE

Average value of a ... over a period, T

$$\bar{P} = I_m^2 R \left\langle \frac{1-\cos 2\omega t}{2} \right\rangle$$

$$\bar{P} = I_m^2 R \times \frac{1}{2}$$

* $$\boxed{\bar{P} = \frac{1}{2} I_m^2 R}$$ A.C. POWER

* $$\boxed{P = I^2 R}$$ D.C. POWER

D.C Power → A.C. Power

$$I^2 \cancel{R} = \frac{1}{2} I_m^2 \cancel{R}$$

$$I^2 = \frac{I_m^2}{2}$$

Taking $\sqrt{\ }$ on both sides

$$\sqrt{I^2} = \sqrt{\frac{I_m^2}{2}}$$

I_{rms} — $$\boxed{I = \frac{I_m}{\sqrt{2}}}$$

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$$

NOTE

Average value of a function over a period, T is given by

$$\langle F(t) \rangle = \frac{1}{T}\int_0^T F(t)\,dt$$

$$\langle \cos 2\omega t \rangle = \frac{1}{T}\int_0^T \cos 2\omega t\, dt$$

$$= \frac{1}{T}\left[\frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$\int \cos ax\, dx = \frac{\sin ax}{a}$$

$$= \frac{1}{2\omega T}\left[\underbrace{\sin 2\omega T}_{0} - 0\right]$$

$$= \frac{1}{2\omega T} \times 0 = 0$$

$$\therefore \langle \cos 2\omega t \rangle = 0$$

I or I_{rms}

= effective current

= root mean square value of current

$$I_{rms} = \frac{I_m}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}\cdot I_m}{2} = \frac{1.414 \cdot I_m}{2}$$

$$\boxed{I \text{ (or) } I_{rms} = 0.707 I_m}$$

⇒ $$I = \sqrt{I} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

Average power ($\bar{P}$)

$$\bar{P} = \frac{1}{2} I_m^2 R = I^2 R$$

$$V = \frac{V_m}{\ } = 0.707 V_m$$

$$V = IR$$

$$I = \frac{V}{R}$$

$$\frac{1}{2}\frac{V_m^2}{\cancel{R^2}} \cdot \cancel{R} = \frac{V^2}{\cancel{R^2}} \cdot \cancel{R}$$

$$V^2 = \frac{V_m^2}{2}$$

$$V = V_m$$

$$\boxed{V = \frac{V_m}{\sqrt{2}} = 0.707 V_m}$$

$V_m = I_m R$

$$\frac{V_m}{\sqrt{2}} = \frac{I_m}{\sqrt{2}} R$$

$$\boxed{V = IR}$$

V = rms voltage (or) effective voltage

$$V = \frac{V_m}{\sqrt{2}}$$

$V_m = \sqrt{2}\, V$

$$\boxed{V_m = 1.414\, V}$$

NOTE: When rms voltage is 220 V, the peak voltage is given by

$$V_m = 1.414 \times 220 = \underline{\underline{311 \text{ volt}}}$$